Numerical Methods: Class Test 1 Solution

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Problem 1

Use bisection method to approximate the root of the equation $e^x = 3x$. Do 5 iterations.

Solution

The given equation can be written as $e^x - 3x = 0$. Let $f(x) = e^x - 3x$. In order to apply the bisection method, we first need to find a and b such that $f(a) \cdot f(b) < 0$. Note that

$$f(0) = 1 \text{ and } f(1) = e - 3 < 0.$$

Thus, the equation has a root between 0 and 1. Now we will apply the bisection method by taking the interval $[0, 1]$.

- **First iteration:**

$$x_1 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5.$$

Since $f(0.5) = 0.14872 > 0$, for the next iteraiton, we have $a = 0.5$ and $b = 1$.

- **Second iteration:**

$$x_2 = \frac{a+b}{2} = \frac{0.5+1}{2} = 0.75.$$

Since $f(0.75) = -0.13299 < 0$, for the next iteraiton, we have $a = 0.5$ and $b = 0.75$.

- **Third iteration:**

$$x_3 = \frac{a+b}{2} = \frac{0.5+0.75}{2} = 0.625.$$

Since $f(0.625) < 0$, for the next iteraiton, we have $a = 0.5$ and $b = 0.625$.

- **Fourth iteration:**

$$x_4 = \frac{a+b}{2} = \frac{0.5+0.625}{2} = 0.5625.$$

Since $f(0.5625) > 0$, for the next iteraiton, we have $a = 0.5625$ and $b = 0.625$.

- **Fifth iteration:**

$$x_5 = \frac{a+b}{2} = \frac{0.5625+0.625}{2} = 0.59375.$$

Since $f(0.59375) > 0$, for the next iteraiton, we have $a = 0.59375$ and $b = 0.625$.

Therefore, the root of the equation after 5 iterations is 0.59375.

To summarize this, the following table is useful.

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
0	1	0.5	+ve	-ve	+ve
0.5	1	0.75	+ve	-ve	-ve
0.5	0.75	0.625	+ve	-ve	-ve
0.5	0.625	0.5625	+ve	-ve	+ve
0.5625	0.625	0.29375	+ve	-ve	+ve

Problem 2

Using the regula falsi method, compute a real root of the equation $3x \sin x = e^x$. Do 5 iterations.

Solution

The given equation can be written as $3x \sin x - e^x = 0$. Let $f(x) = 3x \sin x - e^x$. In order to apply the regula falsi method, we first need to find a and b such that $f(a) \cdot f(b) < 0$. All inputs are in radian (*this is important, as if you change to degree, your value will be different. In your calculator, make sure that you are using the radian value. If you are using the calculator* `fx 991ES PLUS`*, then on the top of the display you can see* `R` *or* `D`. `R` *stands for radian and* `D` *stands for degree. In order to change to radian follow the following operations:* `SHIFT -> SETUP`*, then option 3 is* `Deg` *and option 4 is* `Rad`. *Make sure you choose option* `4`). Note that

$$f(0) = -1, \qquad f(1) = -0.193868,$$
$$f(-1) = 2.156533.$$

Thus, we take $a = -1$ and $b = 0$.

- **First iteration:** $a = -1$ and $b = 0$.

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = -0.316803.$$

Since $f(x_1) < 0$, for the next iteraiton, we have $a = -1$ and $b = -0.316803$.

- **Second iteration:** $a = -1$ and $b = -0.316803$.

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = -0.430908.$$

Since $f(x_2) < 0$, for the next iteraiton, we have $a = -1$ and $b = -0.430908$.

- **Third iteration:** $a = -1$ and $b = -0.430908$.

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$
$$= -0.458516.$$

Since $f(x_3) < 0$, for the next iteraiton, we have $a = -1$ and $b = -0.458516$.

- **Fourth iteration:** $a = -1$ and $b = -0.458516$.

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$
$$= -0.464323.$$

Since $f(x_4) < 0$, for the next iteraiton, we have $a = -1$ and $b = -0.464323$.

- **Fifth iteration:** $a = -1$ and $b = -0.464323$.

$$x_5 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$
$$= -0.465504.$$

Since $f(x_5) < 0$, for the next iteraiton, we have $a = -1$ and $b = -0.465504$.

Therefore, the root of the given equation after 5 iterations is -0.465504. **YOUR ANSWER COULD BE DIFFERENT. IT DEPENDS WHAT ARE THE INITIAL VALUES YOU HAVE CHOOSEN**

The following table summarizes the computation.

a	b	$c = \frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(a)$	$f(b)$	$f(c)$
-1	0	-0.316803	+ve	-ve	-ve
-1	-0.316803	-0.430908	+ve	-ve	-ve
-1	-0.430908	-0.458516	+ve	-ve	-ve
-1	-0.458516	-0.464323	+ve	-ve	-ve
-1	-0.464323	-0.465504	+ve	-ve	-ve

Problem 3

Use Newton-Raphson method to evaluate $\sqrt{13}$ correct upto 4 decimal places.

Solution

We want to find the value of $\sqrt{13}$ by Newton-Raphson's method. For that take the function $f(x) = x^2 - 13$. Clearly, one of the root of the equation $f(x) = 0$ is $\sqrt{13}$. We will approximate this using Newton-Raphson method. We have

$$f(x) = x^2 - 13 \text{ and } f'(x) = 2x.$$

Thus, newton-Raphson iteration scheme will be

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \\ &= x_n - \frac{x_n^2 - 13}{2x_n}, \quad n \geq 0. \end{aligned}$$

We will start with an inital guess as $x_0 = 1$. Since $f'(1) = 2 \neq 0$, this guess is a valid guess.

- **First iteration:** $x_0 = 1$.

$$\begin{aligned} x_1 &= x_0 - \frac{x_0^2 - 13}{2x_0} \\ &= 7. \end{aligned}$$

- **Second iteration:** $x_1 = 7$.

$$\begin{aligned} x_2 &= x_1 - \frac{x_1^2 - 13}{2x_1} \\ &= 4.4285714. \end{aligned}$$

- **Third iteration:** $x_2 = 4.4285714$.

$$\begin{aligned} x_3 &= x_2 - \frac{x_2^2 - 13}{2x_2} \\ &= 3.6820276. \end{aligned}$$

- **Fourth iteration:** $x_3 = 3.6820276$.

$$\begin{aligned} x_4 &= x_3 - \frac{x_3^2 - 13}{2x_3} \\ &= 3.606345. \end{aligned}$$

- **Fifth iteration:** $x_4 = 3.606345$.

$$\begin{aligned} x_5 &= x_4 - \frac{x_4^2 - 13}{2x_4} \\ &= 3.605551. \end{aligned}$$

- **Sixth iteration:** $x_5 = 3.605551$.

$$x_6 = x_5 - \frac{x_5^2 - 13}{2x_5}$$
$$= 3.605551.$$

Thus, the value $\sqrt{13}$ correct to 4 decimal places is 3.6055.