MATRIX GROUPS

(MTH565)

Quiz 8

Thursday, 13th November 2025

Name: ______________________

Roll Number: ______________________

Obtained Marks: ________________ **/10**

EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. The total point for the problems is 11, but the maximum obtainable score is 10.
6. **Duration:** 30 minutes.

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Good Luck!

Problem Set

Problem 1

Let G_1 and G_2 be two matrix groups with Lie algebras $\mathfrak{g}_1$ and $\mathfrak{g}_2$, respectively. We have seen that if $f : G_1 \to G_2$ is a smooth homomorphism, then the differential map $df_I : \mathfrak{g}_1 \to \mathfrak{g}_2$ is a Lie algebra homomorphism.

(i) Show that for any $a \in G_1$ and $B \in \mathfrak{g}_2$, we have

$$df_I(\mathrm{Ad}_a(B)) = \mathrm{Ad}_{f(a)}(df_I(B)).$$

(ii) Use the above result to show that if f is an isomorphism, then df_I is a Lie algebra isomorphism.

$\boxed{2+2}$

Problem 2

Let G be a matrix group and $U \in G$. Show that each of the function is differentiable and determine its derivative at I.

(i) $L_U : G \to G, \quad A \mapsto UA,$

(ii) $R_U : G \to G, \quad A \mapsto AU,$

(iii) $C_U : G \to G, \quad A \mapsto UAU^{-1}.$

$\boxed{1+1+2}$

Problem 3

It is given that for $A = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}$, the exponential is

$$\exp(A) = \begin{pmatrix} \cosh a & \sinh a \\ \sinh a & \cosh a \end{pmatrix}.$$

Find the Lie algebra of the matrix group

$$G = \left\{ A \in GL_2(\mathbb{R}) : A^T Q A = Q \right\}, \quad \text{where } Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$\boxed{3}$

SOLUTION SPACE

Solution (continued)

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