Matrix Groups

(MTH565)

Quiz 5

Thursday, 09th October 2025

Name: ____________________

Roll Number: ____________________

Obtained Marks: ______________ **/10**

EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. The total point for the problems is 12, but the maximum obtainable score is 10.
6. **Duration:** 30 minutes.

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Good Luck!

Problem Set

Problem 1

Recall that the dimension of a matrix group G is the dimension of corresponding Lie algebra $\mathcal{L}(G)$. Find the Lie algebra of $O(n, \mathbb{R})$ and hence deduce the dimension of $O(n, \mathbb{R})$.

$\boxed{3 + 2 = 5}$

Problem 2

For $x \in \mathbb{R}$, consider the matrix

$$A = \begin{pmatrix} 0 & x \\ -x & 0 \end{pmatrix}.$$

Show that $e^A \in SO(2, \mathbb{R})$. (Note that you need to show e^A is a rotation matrix with some rotation angle).

$\boxed{3}$

Problem 3

Prove the following:

1. For any matrix $A \in M_n(\mathbb{K})$, the matrix $e^A \in GL_n(\mathbb{K})$.
2. For any $A, B \in M_n(\mathbb{K})$ with $A \in GL_n(\mathbb{K})$,

$$e^{ABA^{-1}} = Ae^B A^{-1}.$$

$\boxed{2 + 2 = 4}$

You can use the following things:

- For $A(t) \in M_n(\mathbb{K})$,

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} \det A(t) = \operatorname{trace}\left(A'(0)\right).$$

- For any matrix $A \in M_n(\mathbb{K})$,

$$\det e^A = e^{\operatorname{trace}(A)}.$$

SOLUTION SPACE

Solution (continued)

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