MATRIX GROUPS

(MTH565)

Quiz 4

Thursday, 18th September 2025

Name: ______________________________

Roll Number: ______________________________

Obtained Marks: ______________________ **/10**

EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. The total point for the problems is 12, but the maximum obtainable score is 10.
6. **Duration:** 30 minutes.

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Good Luck!

Problem Set

Problem 1

Prove or disprove:

(i) $O(5)$ is isomorphic to $SO(5) \times \{1, -1\}$.

(ii) $O(2)$ is isomorphic to $SO(2) \times \{1, -1\}$.

$\boxed{3 + 2 = 5}$

Problem 2

Define the *Affine group* as

$$\mathrm{Aff}_n(\mathbb{F}) := \left\{ \begin{pmatrix} A & \mathbf{v} \\ 0 & 1 \end{pmatrix} : A \in GL_n(\mathbb{F}) \text{ and } \mathbf{v} \in \mathbb{F}^n \right\}.$$

Given any $X = \begin{pmatrix} A & \mathbf{v} \\ 0 & 1 \end{pmatrix} \in \mathrm{Aff}_n(\mathbb{F})$, we can identify it with a functions $f(\mathbf{x}) = A\mathbf{x} + \mathbf{v}$ from $\mathbb{F}^n$ to $\mathbb{F}^n$. Define a translated line

$$\ell_{\mathbf{v}_0} = \{\mathbf{v}_0 + \mathbf{v} : \mathbf{v} \in W\},$$

where $\mathbf{v}_0 \in \mathbb{F}^n$ and $W \subset \mathbb{F}^n$ is an 1-dimensional $\mathbb{F}$-subspace. Prove that f sends translated lines in $\mathbb{F}^n$ to translated lines in $\mathbb{F}^n$.

$\boxed{3}$

Problem 3

Recall that the translational group is defined as

$$\mathrm{Trans}(\mathbb{R}^n) = \{f \in \mathrm{Isom}(\mathbb{R}^n) : f(\mathbf{x}) = \mathbf{x} + \mathbf{v}, \mathbf{v} \in \mathbb{R}^n\}.$$

(i) Show that $\mathrm{Trans}(\mathbb{R}^n)$ can be thought as a subset of $GL_{n+1}(\mathbb{R})$.

(ii) Assume that $\mathrm{Trans}(\mathbb{R}^n)$ is a subgroup of $\mathrm{Isom}(\mathbb{R}^n)$, show that $\mathrm{Trans}(\mathbb{R}^n)$ is a normal subgroup of $\mathrm{Isom}(\mathbb{R}^n)$.

$\boxed{1 + 3 = 4}$

SOLUTION SPACE

Solution (continued)

Solution (continued)