MATRIX GROUPS

(MTH565)

Quiz 3

Thursday, 4th September 2025

Name: ______________________

Roll Number: ______________________

Obtained Marks: ________________ **/10**

EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. The total point for the problems is 12, but the maximum obtainable score is 10.
6. **Duration:** 30 minutes.

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Good Luck!

Problem Set

Problem 1

True/False problems. If the statement is true, then prove it otherwise provide a counterexample or disprove it.

Let D_r be the set of $n \times n$ real matrices with determinant r.

(i) D_0 is a closed set in $M_n(\mathbb{R})$.

(ii) $GL_n(\mathbb{R})$ is a closed set in $M_n(\mathbb{R})$.

(iii) $\bigcup_{r \in \mathbb{R} \setminus \{0\}} D_r$ is compact in $M_n(\mathbb{R})$.

(iv) $O_n(\mathbb{R})$ is closed in $M_n(\mathbb{R})$.

(v) A continuous function maps a bounded set to bounded set.

$\boxed{1 + 2 + 1 + 2 + 1 = 6}$

Problem 2

Consider the set of orthogonal matrices with real entries, that is, $O_n(\mathbb{R})$. We say that a set $X \subseteq O_n(\mathbb{R})$ is open (closed) in $O_n(\mathbb{R})$ if there exists an open (closed) set $K \subseteq M_n(\mathbb{R})$ such that $X = K \cap O_n(\mathbb{R})$.

(i) Is $SO(n)$ closed in $O_n(\mathbb{R})$?

(ii) Is it open in $O_n(\mathbb{R})$?

$\boxed{2 + 2 = 4}$

Problem 3

Consider the dot product in $\mathbb{R}^n$ defined by

$$\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{n} x_i y_i, \text{ for } \mathbf{x} = (x_1, \ldots, x_n) \text{ and } \mathbf{y} = (y_1, \ldots, y_n) \in \mathbb{R}^n.$$

Prove that a matrix $A \in O_n(\mathbb{R})$ if and only if for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\langle A\mathbf{x}, A\mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle$.

$\boxed{2}$

SOLUTION SPACE

Solution (continued)

Solution (continued)