MATRIX GROUPS

(MTH565)

Quiz 2

Thursday, 28^{th} August 2025

Name: ______________________

Roll Number: ______________________

Obtained Marks: ________________ **/10**

EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. The total point for the problems is 11, but the maximum obtainable score is 10.
6. **Duration:** 30 minutes.

- -

Good Luck!

Problem Set

Problem 1

(i) Determine the groups $GL_1(\mathbb{C}), SL_1(\mathbb{C}), O_1(\mathbb{C})$ and $SO_1(\mathbb{C})$.

$\boxed{0.5 \times 4 = 2}$

(ii) Find the inverse of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 3 \end{bmatrix}$ in $GL_2(\mathbb{Z}_{11})$.

$\boxed{2}$

Problem 2

(i) Prove that $SU(2) = \left\{ \begin{bmatrix} a & b \\ -\bar{b} & \bar{a} \end{bmatrix} : a, b \in \mathbb{C} \text{ and } |a|^2 + |b|^2 = 1 \right\}$.

$\boxed{2}$

(ii) Show that $SO(3)$ is a normal subgroup of $O(3)$. Further identify the quotient group $O(3)/SO(3)$ (you need to prove that which space this quotient group is isomorphic to).

$\boxed{1 + 2 = 3}$

Problem 3

(i) Is $SL_2(\mathbb{Z})$ a subgroup of $GL_2(\mathbb{R})$ with usual matrix multiplication operation?

$\boxed{1}$

(ii) Write all the elements of $O_2(\mathbb{Z})$ and $SO_2(\mathbb{Z})$.

$\boxed{0.5 \times 2 = 1}$

SOLUTION SPACE

Solution (continued)

Solution (continued)