Matrix Groups: Homework #4

Based on group and field

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Problem 1

In a field $\mathbb{F}$, prove that $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

Problem 2

Let

$$GL(\mathbb{R}^n) = \{T : \mathbb{R}^n \to \mathbb{R}^n : T \text{ is linear and invertible}\}.$$

Show that $GL(\mathbb{R}^n)$ is a group under composition of functions. Can you think of a relation between $GL(\mathbb{R}^n)$ and $GL_n(\mathbb{R})$?

Problem 3

We have seen in the lectures that $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is a rotation matrix (rotation in the counterclockwise direction by an angle θ).

Now, let T_θ denote the reflection about the line L_θ through the origin that makes an angle θ with the x-axis. Write the matrix of T_θ. Check whether the set $\{T_\theta : \theta \in \mathbb{R}\}$ of all such reflections forms a group under matrix multiplication.

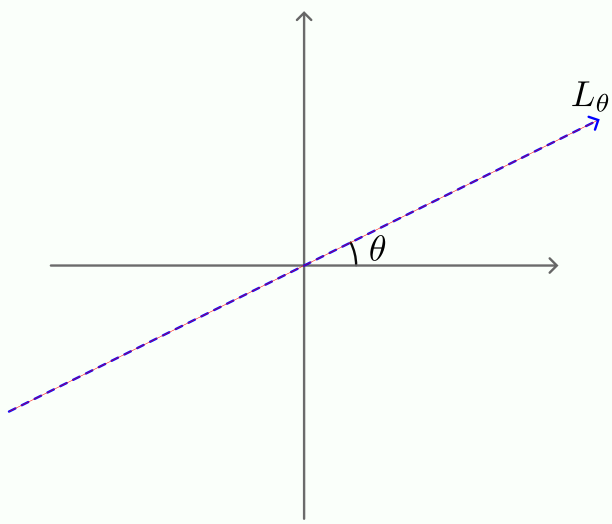

Figure 1: Reflection about the line L_θ

Problem 4

Show that the set of all rotations and reflections (described in the previous problem) is a group under matrix multiplication. Can you identify this group?