Matrix Groups: Homework #1

Based on review of linear algebra

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Theory

Let V, W be vector spaces over $\mathbb{R}$. A map $T : V \to W$ is called a *linear map* if for $\alpha_1, \alpha_2 \in \mathbb{R}$ and $\boldsymbol{v}_1, \boldsymbol{v}_2 \in V$,

$$T(\alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2) = \alpha_1 T(\boldsymbol{v}_1) + \alpha_2 T(\boldsymbol{v}_2).$$

Conventions

1. In all the problem below, unless specified, V, W denotes the vector spaces over $\mathbb{R}$.
2. Linear map and linear transformation will be used interchangeably.

Problem 1

Let T be a linear transformation from V to W. Show that $T(\mathbf{0}) = 0$.

Problem 2

Describe geometrically the action of T on the square whose vertices are at $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$.

1. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$
2. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$
3. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \end{pmatrix}$
4. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \end{pmatrix}$

Problem 3

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y \\ 3x - z \\ 2y - 3z \\ 4x + 2y + z \end{pmatrix}.$$

Find the matrix of T with respect to the standard bases. Also find the matrix of T with respect to the bases

$$\{e_1 + e_2, e_1 - e_2, e_3\} \text{ and } \{e_1, e_2, e_3, e_4\}.$$