Limits

Engineering Mathematics-I

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Calculus for single variable

- Intermediate Form

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- L'Hospital Rule

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- Improper integral

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- Intermediate Form
- L'Hospital Rule
- Rolle's Theorem
- Mean Value Theorem
- Expansion of function
- Taylor and Maclaurin series
- Riemann Integration
- Riemann Sum
- Improper integral
- Beta and Gamma functions and their properties

Today's Goal

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$\frac{\infty}{\infty}$	$\lim_{x\to\infty} \frac{x^2-1}{x-1}$
$\infty - \infty$	$\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right)$
$0 \times \infty$	$\lim_{x\to\infty} \frac{2x}{x^3-1} \cdot \ln x$

Intermediate Form Cont...

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1^∞	$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$
∞^0	$\lim_{x \to \frac{\pi}{2}} (\tan x)^{x - \frac{\pi}{2}}$

Limit of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

For these types of limits, we will use L'Hospital Rule.

Theorem (L'Hospital Rule)

For a limit $\lim_{x \to a} \frac{f(x)}{g(x)}$ *of the indeterminate form* $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ *exists or equals to* $\pm\infty$.

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4. $\lim_{x \to 1} \dfrac{x^3 - x^2 \ln x + \ln x - 1}{x^2 - 1}$

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5. If $\lim_{x\to\infty}\left(\dfrac{x^2-1}{x+1}-ax-b\right)=0$, then find the values of a and b.

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5. If $\lim_{x\to\infty} \left(\frac{x^2-1}{x+1} - ax - b\right) = 0$, then find the values of a and b.

6. $\lim_{t\to 0} \left(t + \frac{1}{t}\right)\left((4-t)^{3/2} - 8\right)$

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6. $\lim_{x \to 0} 2x \tan\left(\frac{\pi}{2} - x\right)$

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&= \lim_{x\to\infty}\frac{x^2 - (x^2 + x)}{\left(x + \sqrt{x^2 + x}\right)} = \lim_{x\to\infty}\frac{-x}{x + \sqrt{x^2 + x}}
\end{aligned}$$

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$$=\lim_{x\to\infty}\frac{-x/x}{x/x+(\sqrt{x^2+x})/x}$$

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&=\lim_{x\to\infty}\frac{x^2-(x^2+x)}{\left(x+\sqrt{x^2+x}\right)}=\lim_{x\to\infty}\frac{-x}{x+\sqrt{x^2+x}}\\
&=\lim_{x\to\infty}\frac{-x/x}{x/x+\left(\sqrt{x^2+x}\right)/x}=-\frac{1}{1+1}=-\frac{1}{2}
\end{aligned}$$

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$$\lim_{x\to a} f(x) = 1 \text{ and } \lim_{x\to a} g(x) = \infty,$$

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