Engineering Mathematics-1: Homework #2

Based on application of derivative

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Instruction

This set of problems is based on Rolle's theorem and Lagrange's mean value theorem. Let us recall both the theorems (for details see in the lecture notes).

Rolle's Theorem

Let f be defined on $[a, b]$ such that

- f is continuous on $[a, b]$,
- f is differentiable on (a, b) and
- $f(a) = f(b)$.

Then, there exists $c \in (a, b)$ such that

$$f'(c) = 0.$$

Lagrange's Mean Value Theorem

Let f be defined on $[a, b]$ such that

- f is continuous on $[a, b]$,
- f is differentiable on (a, b) and

Then, there exists $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

Problem 1

Verify Rolle's theorem for the function x^2 in $(-1, 1)$.

Problem 2

Verify Rolle's theorem for $f(x) = \log\left(\frac{x^2-6}{x}\right)$ in the interval $[-2, 3]$.

Problem 3

For the following function find c using mean value theorem.

$$f(x) = (x-1)(x-2)(x-3), \quad \text{in the interval } [0, 4].$$

Problem 4

Show that if $f'(x) = 0$ at each point in the interval (a, b), then $f(x)$ is constant over (a, b).

Problem 5

Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function.

$$f(x) = x^3 + 2x^2 - x \quad \text{on} \quad [-1, 2].$$

Problem 6

Determine if the Mean Value Theorem can be applied to the following function on the the given closed interval. If so, find all possible values of c.

$$f(x) = x + 3\cos x \quad \text{on} \quad [-\pi, \pi].$$

Problem 7

Check the validity of Lagrange's mean value theorem for the function

$$f(x) = x^2 - 3x + 5$$

on the interval $[1, 4]$. If the theorem holds, find a point c satisfying the conditions of the theorem.