

Engineering Mathematics-I: Homework #1

Based on Statements

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Instruction

We recall the statement and how to write a statement using mathematical quantifiers. For example, consider the following statement

Every student of this class is from the computer science branch.

We want to write the above statement using the quantifiers (*for all* \forall and *there exists* \exists) and then we will write the negation.

Let

\mathcal{S} = set of all students of this class.

Then we can write the above statement using quantifiers as

$\forall s \in \mathcal{S}$ (*s is from the computer science branch.*)

The negation of the above statement will be as follows.

There is a student of this class which is not from the computer science branch.

In terms of quantifiers,

$\exists s \in \mathcal{S}$ (*s is not from the computer science branch.*)

Problem 1

Write the negation of the following statements.

1. 5 is a prime number.
2. Moscow is the capital of China.
3. The literacy rate in India has increased since 1947.
4. The child is cute.
5. The market will not be open in the evening.
6. The square of the integer n is divisible by the prime p .
7. $x^2 + 3 = 27$.

Problem 2

Identify the quantifiers.

1. There is a rotten apple in the basket of apples.

In this example, the quantifier is *there exists*, that is, \exists .

2. All students of SPNREC Araria are from Bihar.
3. Every classroom has a smartboard.
4. Every page in this book contains at least 500 words.
5. All tables in the room are dirty.
6. There exists a student in the classroom who is at least 6 feet tall.
7. In this book some pages do not contain any picture.
8. I can find a millionaire in this room.
9. Every student in the classroom is at least 5 feet tall.
10. There is a student who knows how to speak German.

Problem 3

Write the following statements using quantifiers and write the negation in the plain English. Finally, write the negation in terms of quantifiers. The first example is solved. I will write negation in red color.

1. There is a rotten apple in the basket of apples.

Let \mathcal{A} be the set of all apples in the basket.

- $\exists a \in \mathcal{A}$ (a is rotten).
- For any apple in the basket, the apple is not rotten.
- $\forall a \in \mathcal{A}$ (a is not rotten).

2. All students of SPNREC Araria are from Bihar.
3. Every classroom has a smartboard.
4. Every page in this book contains at least 500 words.
5. All tables in the room are dirty.
6. There exists a student in the classroom who is at least 6 feet tall.
7. In this book some pages do not contain any picture.
8. Given any book on the table, it contains a preface.
9. There is a fuse tubelight in the classroom.
10. Each lady in this room is either intelligent or beautiful.
11. There is a student in the class who is hardworking or intelligent.
12. Every student in the class is hardworking or intelligent.

Problem 4

Write the negation of the following statements. The first example is shown.

1. The boy is intelligent and handsome.

- The boy is either not intelligent or not handsome.

This statement can also be seen as The boy is intelligent and the boy is handsome and the negation can be written as The boy is not intelligent or the boy is not handsome.

2. Sumit and Amit both are intelligent.

3. The classroom has a white board and a black board.

4. Calculus and Linear Algebra both are in the syllabus.

5. Either India or South Africa will make to the final.

6. The book belongs to either Vikas or Nitin.

7. Neither my father nor I have studied History.

8. $a \leq b$.

Problem 5

Write the negation of the following statements.

1. If the apple is red, then it is ripe.

2. If I will go to the school, I will get a sweet.

3. The boy is rich, if he owns a BMW car.

4. The two boys stay in the same house, if they are brothers.

5. The mobile handset has a camera implies that it has Wi-Fi hot-spot.

6. If the student is good in mathematics, then he is humble.

7. If it will rain, then I will not go to school.