

Complex Variables: Homework #3

Based on complex functions

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Problem 1

In the following problems represent $f(z) = u(x, y) + \iota v(x, y)$. Let me show an example. If $f(z) = z^2$, then put $z = x + \iota y$. We have

$$\begin{aligned} f(z) = z^2 &= (x + \iota y)^2 \\ &= x^2 + (\iota y)^2 + 2x(\iota y) \\ &= x^2 - y^2 + \iota(2xy). \end{aligned}$$

Thus,

$$u(x, y) = x^2 - y^2 \quad \text{and} \quad v(x, y) = 2xy.$$

1. $f(z) = z^3$
2. $f(z) = 5z^2 - 12z + 3 + 2\iota$
3. $f(z) = \frac{z-2}{z+2}$
4. $f(z) = \frac{\text{Re}(z^2)}{|z|}$, where $\text{Re}(z^2)$ represents the real part of z^2 .
5. $f(z) = |z|^2$
6. $f(z) = z\bar{z}^2 + \bar{z}$
7. $f(z) = z\bar{z}$
8. $f(z) = |z|^2 + \bar{z}^2 + z^2$
9. $f(z) = z^3 + \iota\bar{z} + 1 + \iota$
10. $f(z) = z^4 - (1 + \iota)\bar{z}^2 - (1 - \iota)|z|^2$

Problem 2

If $f(z) = u(x, y) + \iota v(x, y)$, then the real part of f is $u(x, y)$ and the imaginary part of f is $v(x, y)$. In the following problems find $\text{Re}(f)$, and $\text{Im}(f)$ and their values at the given point z .

1. $f(z) = 5z^2 - 12z + 3 + 2\iota$ at $z = 4 - 3\iota$
2. $f(z) = \frac{1}{1-z}$ at $z = 1 - \iota$
3. $f(z) = \frac{z-2}{z+2}$ at $z = 8\iota$
4. $f(z) = \frac{z-\iota}{z+\iota}$ at $z = 1$
5. $f(z) = (6 - 5\iota)z + 1 - 3i$ at $z = 1 + \iota$